MODELLING ULTRASONIC ARRAY PERFORMANCE IN SIMPLE STRUCTURES

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1 INTRODUCTION

Ultrasonic phased array transducers have been available for more than two decades, they are widely used in medical imaging [7][9][10]. In non-destructive evaluation (NDE), phased array transducers have been used in a wide range of applications including inspection of nuclear [8]. Using of phased array transducers mechanical scanning can be decrease and the overall inspection time reduced. The ultrasonic beam can be steered and focused on particular destination by controlling the firing sequence of the individual elements.

A linear phased array transducer consists of multiple piezoelectric elements separated by a finite distance. The ultrasonic beam emitted can be steered and focused by selection of the time delays on individual element excitation. This steering and focusing behaviour is an important issue for applying linear phased array transducers into non-destructive testing inspections [2]. This paper aims to understand the behaviour of linear phased arrays transducers and develop methods of optimising their performance [3]. The most common method to investigate their behaviour is by computer modelling. Due to the limitation of computer performance, most authors employ a two-dimensional computer model to investigate their behaviour [3][4]. Some authors employ a three-dimensional computer model [5].

These modelling results generally conclude that a linear phased array transducer with a large element width has reduced performance, and that the critical width is $\lambda/2$ [31][6]. This would limit the aperture of inspections, therefore many researches are going on to improve the aperture size [4]. In this paper, the focusing performance of linear phased array transducer is investigated by three-dimensional computer model by Huygens' principle and a method of improving the focusing performance of linear phased array transducers is developed.

2 PHASED ARRAY FOCUSED BY GENERALISED FOCUSING EQUATION

2.1 SIMULATION METHOD

2.1.1 METHOD BY USING A 3D MODEL

A linear phased array transducer, is constructed of a series of array elements. These elements can be configured to fire as individual transducers, that is to say it can be fired as different pressure and different phase. Therefore, an array transducer surface can be modeled as a series of plane rectangular transducers with the width equal to the element width and the height equal to the element height, the number of the plane rectangular transducers being equal to the number of elements of the array transducer. The ultrasonic field of an array transducer is then constructed by the vectorial sum of the ultrasonic fields of its elements [5].

To model the array element, according to Huygens' principle, elements can discretised into emitting zones. According to Krautkramer, the pressure and the phase angle at any point in the output field can be calculated by equation 1 [32]

$$p = \frac{p_s}{a\lambda} \tag{1}$$

Where, p is the acoustic pressure at a point at distance *a* from a reference point having pressure p_s and λ is the wavelength.

2.1.2 FITTING THE GFE TO 3D MODEL

The generalized focusing equation (GFE) is the most common algorithm used to focus and steer the ultrasonic beam from linear phased array transducers. The beam is focused or is steered by adjusting the firing time or phasing of each individual element and the firing time of each element can be calculated by applying the GFE.

According to Azar et al, The principle of the GFE is based on the geometry of the linear phased array transducer and the location of the focal point as shown in figure 1. By assuming that the emitted wave of each element is spherical wave, the following relationship can be obtained: [2][7][9]

$$t_n = \frac{F}{c}\left\{ \begin{array}{l} \left[1+\left(\frac{\overline{N}d}{F}\right)^2+\frac{2\overline{N}d}{F}\sin\theta_s\right]^{1/2} \\ -\left[1+\left(\frac{(n-\overline{N})d}{F}\right)^2-\frac{2(n-\overline{N})d}{F}\sin\theta_s\right]^{1/2} \end{array} \right\} \tag{2}$$

for any number of element N, where t_n is the required time delay for element n = 0, …, N-1, $\overline{N}$ = (N-1)/2, d the center-to-center spacing between elements, F the focal length from the center of the array, θ_s the steering angle from the center of array, and c is the velocity of wave.

A computer model can be configured to take into account the firing delays generated by the GFE to steer and focus the ultrasonic beam of linear phased array transducers. Therefore the performance of transducers focused by GFE and be investigated.

2.2 SIMULATION RESULT

A Huygens' computer model of an array transducer has been constructed. Sets of results in which the firing delays were set by the generalised focusing equation were obtained and used to analyse the performance of the equation and various array configurations. By fixing some common parameters and varying the parameters focal distance, array width, and number of elements, the focusing effects can be compared. For this study the fixed parameters used are shown in table 1

Table 1 Fixed parameters for the testing sample

Velocity of wave, c (m/s)	1500 (water)
Frequency, f (MHz)	7.5
Transducer height (mm)	3
Unit of received emitted field	300x255 & 1000x75
Dimension of emitted field elements (mm)	0.5x0.5 & 0.1x0.1

In figure 2, the focusing performance of a 64 element array with 50mm width array transducers focusing on 30mm, 60mm and 90mm is shown, the array transducer shows a clear focusing on 90mm. However, when it is focused on 30mm and 60mm, there is no single peak, but a double peak is observed at the focal point.

In figure 3, the focusing performance of 64 element array transducers focusing on 60 mm away from transducer is shown. The array transducer with 25mm width shows a clear focusing on 60 mm. However, for 100mm width and 50mm width array transducer, there is no single peak, but again a double peak effect is observed at the focal point.

In figure 4, the focusing performance of 50mm width array transducers focusing on 60 mm is shown. The array transducer with 128 elements shows a clear focusing on 60 mm. However, for 16, 32, 64 elements array transducer, there is no single peak, and a double peak observed.

2.3 DISCUSSION OF RESULTS

As can be seen in figure 4, when the number of elements increases, the pressure on the focal spot goes up. Also, as it is shown in the figure 2, when the focal length increases, the pressure on the focal spot decreases. Furthermore, as it is shown in figure 3, as the array width increases, the pressure on the focal spot increases, provided that there is only a single peak. Pressure on the focal spot has a direct relationship with number of elements and array width, and inverse relationship with focal length, this results agree with the investigation on the influence of phased array element size by Wooh et al [6]. The higher the pressure on the focal spot relative to other points on the field, the better the focusing, and the better the signal to noise ratio at the focus.

As it is shown in the figures 2, 3 and 4, there is often a distortion effect of the peak at the focal point. This effect is more significant when either having a relatively few number of elements, as it is shown in the figure 3, or having a relatively large array width, as it is shown in the figure 4. The effect is also more significant, when the focal length is shorter, as it is shown in figure 2. Furthermore, the distortion can be seen to occur at the focal spot, and the peak is often spilt into two (or more) parts. In other words, multiple peaks appear and having their centre at the focal spot. This not only significantly reduces the pressure on the focal spot, but also could generate a false second echo in an ultrasonic test.

3 THEORETICAL MAXIMUM PERFORMANCE OF ARRAY

As discussed in section 2B, there is a double peak effect at the focal point for some array configurations. This section investigates the question of whether the double peak effect is a natural limitation for those cases or there is a possibility of reducing it? In order to find the answer to this question, the theoretical maximum performance (TMP) of array transducers was investigated.

3.1 METHOD OF CALCULATING THE TMP

3.1.1 CONTRIBUTION OF INDIVIAL ARRAY ELEMENT TO THE FOCUAL SPOT

To investigate the maximum performance of array transducers, the focusing mechanism is the first thing to consider. Array transducers are a series of piezoelectric element. Furthermore, the output sound field is constructed by the summation of every single elements in the array as shown in figure 5. In other words, the pressure on any location in the sound field is the vectorial sum of the pressure at this point due to every element.

To further consider the mechanism of focusing, the pressure construction of P_f in figure 5 is considered. As P_f is equal to the vectorial sum of the pressure at point f due to every element in the array, that is to say, P_f is equal to the vectorial summation of $\overrightarrow{pf_1}$, $\overrightarrow{pf_2}$, $\overrightarrow{pf_3}$, and $\overrightarrow{pf_4}$ in figure 5.

Therefore :-

$$\mathrm{Im}\left(\overrightarrow{P_f}\right) = \left|\overrightarrow{pf_1}\right|\cos\angle\overrightarrow{pf_1} + \left|\overrightarrow{pf_2}\right|\cos\angle\overrightarrow{pf_2} + \left|\overrightarrow{pf_3}\right|\cos\angle\overrightarrow{pf_3} + \left|\overrightarrow{pf_4}\right|\cos\angle\overrightarrow{pf_4}$$

$$= \sum_{i=1}^{4}\left|\overrightarrow{pf_i}\right|\cos\angle\overrightarrow{pf_i}$$

$$\mathrm{Re}\left(\overrightarrow{P_f}\right) = \left|\overrightarrow{pf_1}\right|\sin\angle\overrightarrow{pf_1} + \left|\overrightarrow{pf_2}\right|\sin\angle\overrightarrow{pf_2} + \left|\overrightarrow{pf_3}\right|\sin\angle\overrightarrow{pf_3} + \left|\overrightarrow{pf_4}\right|\sin\angle\overrightarrow{pf_4}$$

$$= \sum_{i=1}^{4}\left|\overrightarrow{pf_i}\right|\sin\angle\overrightarrow{pf_i}$$

Thus,

$$\left|\overrightarrow{P_f}\right| = \left|P_f\right| = \sqrt{\left(\mathrm{Im}\left(\overrightarrow{P_f}\right)\right)^2 + \left(\mathrm{Re}\left(\overrightarrow{P_f}\right)\right)^2}$$

$$\angle P_f = \tan\frac{\mathrm{Im}(\overrightarrow{P_f})}{\mathrm{Re}(\overrightarrow{P_f})} \qquad \textbf{\textit{(3)}}$$

3.1.2 METHOD OF PREDICTING THE MAXIUM PERFORMANCE

To consider the case of focusing an array transducer, let say if the array transducer is configured to focus on point f, the objective of the focusing mechanism will be to maximize the pressure on point f, P_f. For the *Max* ($\left|P_f\right|$), the ultrasonic wave from all e_1, e_2, e_3, e_4 are in phase when they arrive at point f. It is because phase difference between $\overrightarrow{pf_1}$, $\overrightarrow{pf_2}$, $\overrightarrow{pf_3}$, $\overrightarrow{pf_4}$ could be compensated by the firing delay of each array elements. In this case, $\left|P_f\right|$ is simply equal to the sum of $\left|\overrightarrow{pf_1}\right|$, $\left|\overrightarrow{pf_2}\right|$, $\left|\overrightarrow{pf_3}\right|$, $\left|\overrightarrow{pf_4}\right|$. For the general case: -

$$\left|P_f\right| = \sum_{i=1}^{n}\left|\overrightarrow{pf_i}\right| \qquad (4)$$

for an n element array transducer. Therefore, the maximum performance can be predicted by the sum of $\left|\overrightarrow{pf_1}\right|$ to $\left|\overrightarrow{pf_n}\right|$.

3.1.3 MAXIMUM PERFORMENCE MODELLING

To construct a model to predict the maximum performance of an array transducer, the following principles were applied :-

- The principle of ultrasonic filed modeling (section 2)
- The equation of maximum performance (equation 4)

In order to achieve Max ($|P_f|$), firstly, $|\overrightarrow{pf_i}|$ is calculated by using the principle of ultrasonic field modeling with the element e_i modeled as a plane rectangular transducer. Thus, according to the equation of maximum performance, the maximum performance can be found by sum of $|\overrightarrow{pf_i}|$ from i = 1 to n. Therefore, the maximum pressure of the array transducer to point f can be predicted. Furthermore, to achieve more information, series of points are selected and predicted on the center axis of the array transducer, so that a line of maximum performance on the center axis can be plotted.

3.2 RESULT OF THEORITICAL MAXIMUN PERFORMANCE

As discussed in section 3A, the computer model is capable of predicting the line of maximum performance on the center axis of a array transducer. These predictions were made for array transducers of the specifications shown in table 6.1.

Table 3 Selection of array transducer specification for Max. performance examination.

Velocity of wave (c)	1500 m/s
Frequency (f)	7.5 MHz
Transducer Height (mm)	3
Transducer Width (mm)	25, 50, 75, 100
Number of elements	16, 32, 64, 128
Focal Distance(mm)	1 – 200

These result sets are grouped and are investigated in order to compare the effect on the max performance with the change of array width and the number of elements.

It can be seen in figure 6 that when the array width is small, the maximum pressure near the transducer is high, however, a wider transducer has a better maximum performance when the focal distance is longer.

The reason for the strong performance for small transducer at a shorter focal length is because the element width for those transducers are smaller therefore phase pattern is more circular and the capability of focusing is better. However, for a wider transducer, the surface area of the transducer is larger, thus more pressure is possible to transmit to the destination zone. The performance for a wider transducer focus at short focal distance due to non-circular phase pattern but is better when on a long focal distance due to effect of beam spread.

3.3 COMPARING THE MAXIMUM PERFORMENCE WITH THE PERFORMANCE OF GENERALISED FOCUSING EQUATION

As the maximum performance of array transducers has been predicted, it is valuable to compare it with the performance of the array transducer focused by GFE at a particular focal distance. The percentage differences between them are calculated. The measured parameters is the percentage of the underperformance for the GFE in those cases. This is an important measurement to quantify the performance of the GFE.

In figure 7, the case of 64 elements transducers, as the array width is small, the percentage of underperformance is small, that is to say the performance is closer to the max performance that the array transducer is able to achieve. Moreover, for a wider array transducer, it requires a longer focal distance in order to reach the maximum performance of the transducer.

3.4 LIMITATIONS OF THE GENERALISED FOCUSING EQUATION

The maximum performance at a short focal length is higher when the element width is small as shown in figure 7. Therefore, for the situation where the transducer is to be focused at a short focal distance, transducers with small element width have to be used. However, the transducer with a wider array width, has a better performance at a long focusing distance as shown in figure 7. Therefore, for the situation where the transducer is not required to focus on a short focal distance, a wider transducer is a more suitable choice.

At a long focal distance, the GFE is able to configure the transducer to reach its maximum performance. However, for a short focal length, underperformance occurs. For the transducers with a small array element width the underperformance decreases or it reach the maximum achievable performance. That is to say, GFE is good at focusing at a long focal distance. For a short focusing distance it depends on the element width of the array transducer. With a small element width, it is possible to reach the maximum performance with a short focal distance. Therefore, for the situation that requires focusing at a short focal distance, a transducer with small element width is required.

To conclude, there is a limitation for the GFE, a transducer with a wider element cannot be optimized by the GFE, and significant underperformance occurs. The reason for the significant underperformance is the appearance of the distorted peak effect as described in section 2B. This significant underperformance and distorted peak effect are due to the limitation of the GFE but not due to the limitation of the transducer itself. Therefore, it is possible to improve the performance on array transducer focusing by further investigating the focusing logic.

4 FOCUSING CORRECTION

4.1 ANALYSIS THE REASON OF UNDERPERFORMENCE OF THE GFE

As it is concluded in section 3, there is an underperformance for transducers focused by the GFE. In order to investigate whether this underperformance can be improved, the reason of underperformance is investigated.

The GFE was developed by taking the path difference between elements to calculate the time difference between arrivals from every element in the array transducer to the focusing point. Thus, it assumes that the traveling pattern from an element in any direction is constant, that is to say that phase pattern is circular. However, it is only true for point source like element. For a relatively large element compared with the wavelength, traveling pattern is not always circular.

In figure 8, it can be seen that when the element width is small the phase pattern is circular, however, as the width of the element increases the phase pattern becomes less circular than when the element width is small. This can be explained by the interference pattern between different points on the surface of the element. Also, it can been seen that the phase pattern is less circular when near the element, but it becomes more circular further away from the array element. This can be explained by the path different between points on the element surface to a remote part becomes smaller at larger distance from the array element.

As the phase pattern becomes non-circular, the assumption of the GFE becomes invalid. As the non-circularity increases, the difference between the physical paths and the actual phase difference increases, thus, the under performance of the GFE increases. Therefore, the GFE is unable to configure the transducer to reach its best performance in those cases.

By recalling the result from section 3, the underperformance occurs not only when the element width increases but also when the focal length decreases. This is further evidence that the under performance, is due to the non-circular phase pattern.

4.2 ADVANCED FOCUSING ALGORITHM

In order to improve the focusing algorithm, actual phase difference between array elements is investigated. For a point source, the phase pattern is circular, however, for a flat plate with significant width, the phase pattern can be predicted by a Huygens' model. Thus, individual array element can be treated as a plane rectangular transducer with its dimensions equal to the array element. Therefore, phase angle from the element to any remote point can be predicted.

Thus, phase difference for every array element to the focusing point can be predicted by equation 3. In figure 5, elements having the phase different $\angle pf_i$ to a remote point f. This phase difference ($\angle pf_i$) is the actual phase difference between element e_i to the focal point f.

For an n element array transducer, phase difference between every array element to the focusing point can be predicted. The firing delay for the element is equal to phase difference multiplied by the velocity of the ultrasonic wave. Therefore, the firing time for every element can be calculated.

4.3 RESULT FROM THE ADVANCED FOCUSING ALGORITHM

Transducers with the specifications listed in table 4 were simulated, focused by the advanced focusing algorithm(AFA).

Table 4 Selection of array transducer specification for performance examination.

Velocity of wave (c)	1500 m/s
Frequency (f)	7.5 MHz
Transducer Height (mm)	3
Transducer Width (mm)	25, 50, 75, 100
Number of elements	16, 32, 64, 128
Focal Distance(mm)	1 – 180

In figure 9 a transducer with 64 elements and 50mm width is configured to focus on the center axis from 30mm to 180mm. The field patterns are simulated using the Huygens' model to advanced focusing logic and compared with the theoretical maximum performance. It can be clearly seen that in figure 9 the array transducer is able to reach its maximum performance at the desired focal spot and has little distorted peak effect. It can also be seen that, there is a minor change in its field pattern at around half the amplitude of the peak when it is configured to focus near the transducer.

4.4 DISCUSSION OF THE AFA

The key benefit of using the AFA is that, it is able to configure the transducer to focus on the focal spot even when the element width is large or when the focal length is short. With this algorithm, the limitation on the width of the element on the design of a transducer is reduced, and so wider elements can be used. This not only allows the production of cheaper transducers, but also allows wider transducers to be designed. Furthermore, this algorithm also improves the minimum focusing range for a given transducer.

The AFA, firing times depends, not only the geometry of the transducer, but also on the frequency and the velocity of wave. This is because the phase pattern is dependant on both these parameters. However, the GFE only depends on the geometry of the transducer. Therefore, the GFE is much easier to apply to a multiple frequency emission. A typical output from an ultrasonic transducer has a range of frequency components and to one approach to implementing the AFA would be to 'tune' it to the center frequency of the range. Simulation for pulse emission model for array transducer (multiple-frequency emission) has been computed using both the AFA and the GFE in figure 10. The AFA shows an excellent focusing ability at 7.5MHz, however, when the frequency increases or decreases about this point, the focusing ability is reduced. The GFE shows a good focusing ability at low frequency. However, as the frequency increases, the focusing ability decreases.

5 FREQUENCY CORRECTION

In order to improve the AFA a multiple frequency focused algorithm is investigated. This algorithm is the frequency corrected version of the AFA. A pulse in time domain can be represented as serious of frequency components and this conversion in typically achieved using a fast fourier transform(FFT) [33]. The AFA can then be applied to each frequency component in turn. In this way frequency domain can be obtained. By using and inverse FFT the, required pulse for each element to focus an linear phased array can then be obtained.

Transducers focused by the multiple frequency focusing algorithm(MFFA) were simulated by pulse emission model and is shown in figure 11 and figure 12. In figure 11, the pressure alone the central axis of the array transducer is show. The MFFA able to correct the error induced by different in frequencies. The MFFA shows a clear improvement compared with the GFE. A single peak is always be generated by using this algorithm. In figure 12, a cross sectional plot of pressure on the focal point is shown.

6 CONCLUSION

The performances of different configurations of linear array transducer have been investigated. Reasons for under performance in focusing of certain cases has been investigated and the possibility for performance improvement studied. An improved focusing algorithm was tested by a computer simulation based on Huygens' principle and the limitations of the improved algorithm investigated.

Underperformance by GEF of large array element transducers were noticed, thus reason for the under performance was investigated. The solution to the under performance was to correct the timing delays to take account of the field patterns of the elements of the array. This approach was very successful for both single frequency and muti-frequency emission. MFFA could be applied to correct the focus of the ultrasonic beam from large element array transducer and was able to configure the transducer running on its best performance.

References

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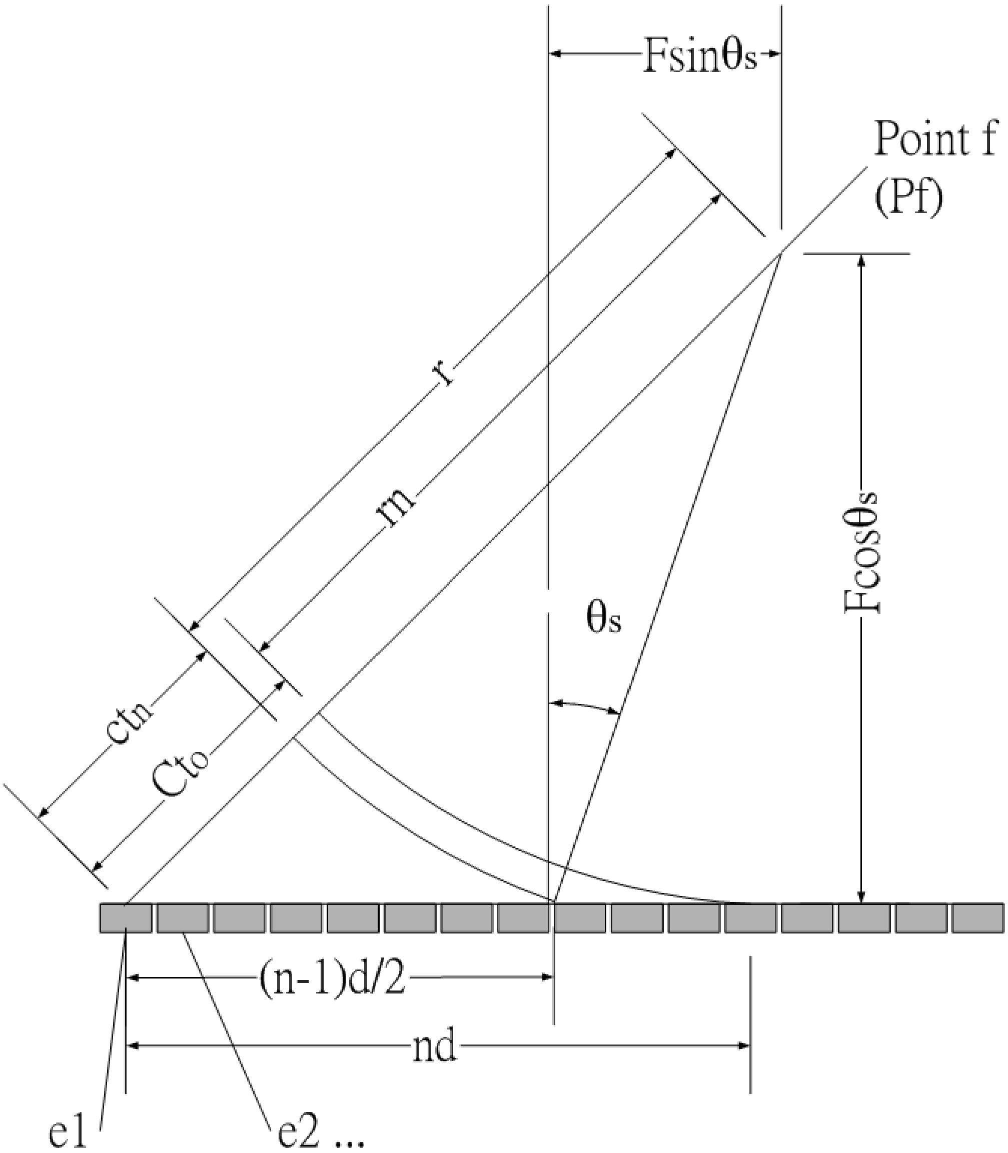


Figure 1 Geometry of linear phased array in deriving the focusing equation.

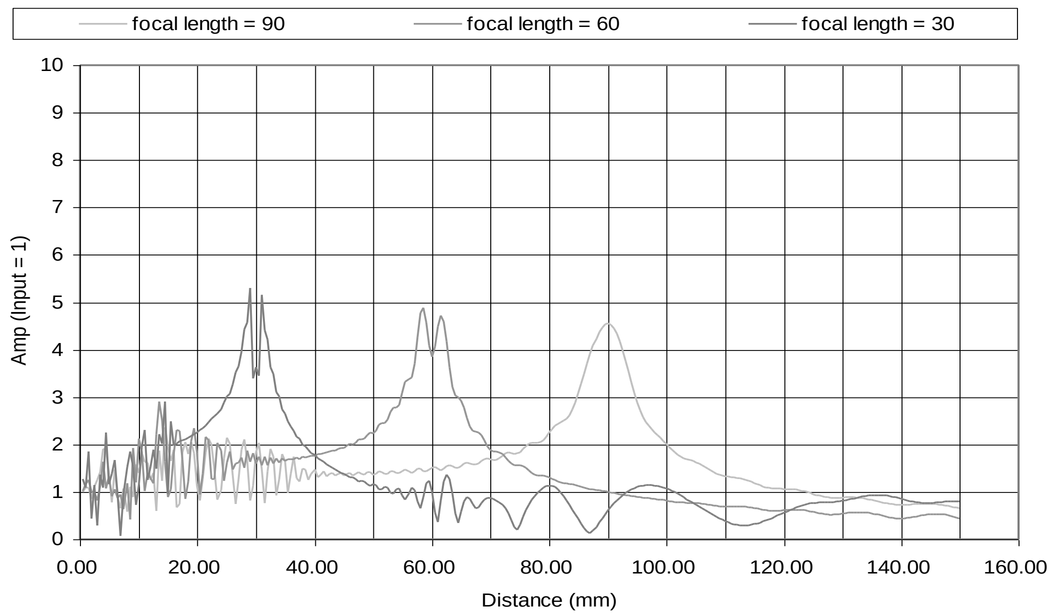


Figure 2 Focusing performance of a 64 element array with a width of 50 mm for 3 different focal lengths.

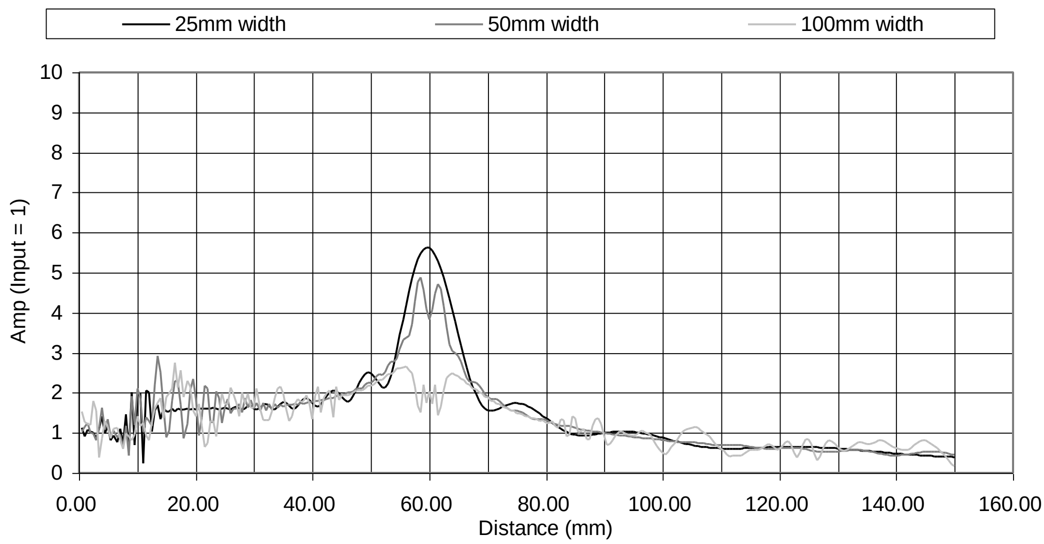


Figure 3 Focusing performance of a 64 element array with for 60 focal length with different element width.

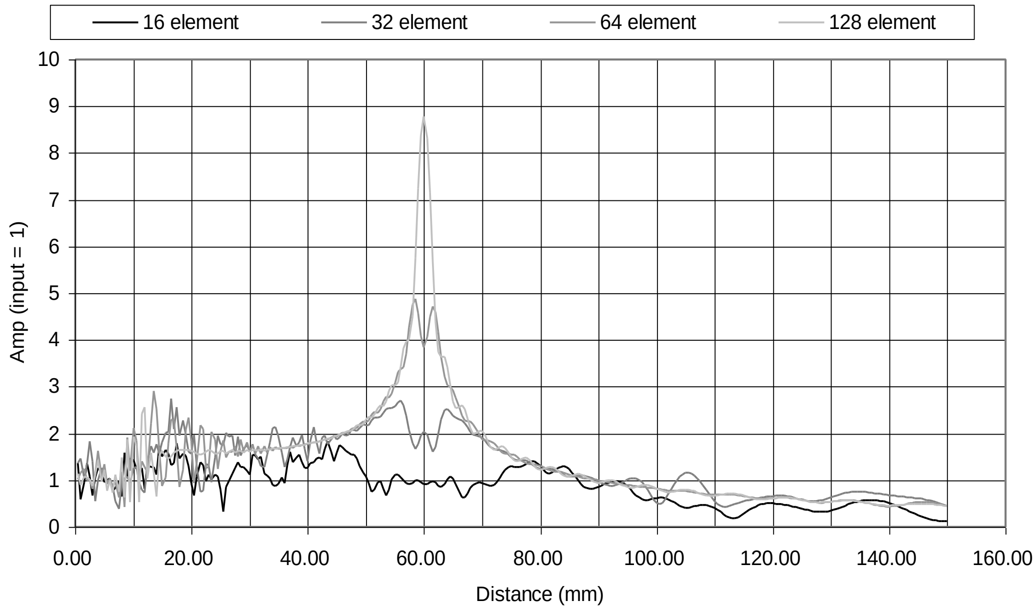


Figure 4 Focusing performance of a array with a width of 50 mm for 60 focal length with different number of element.

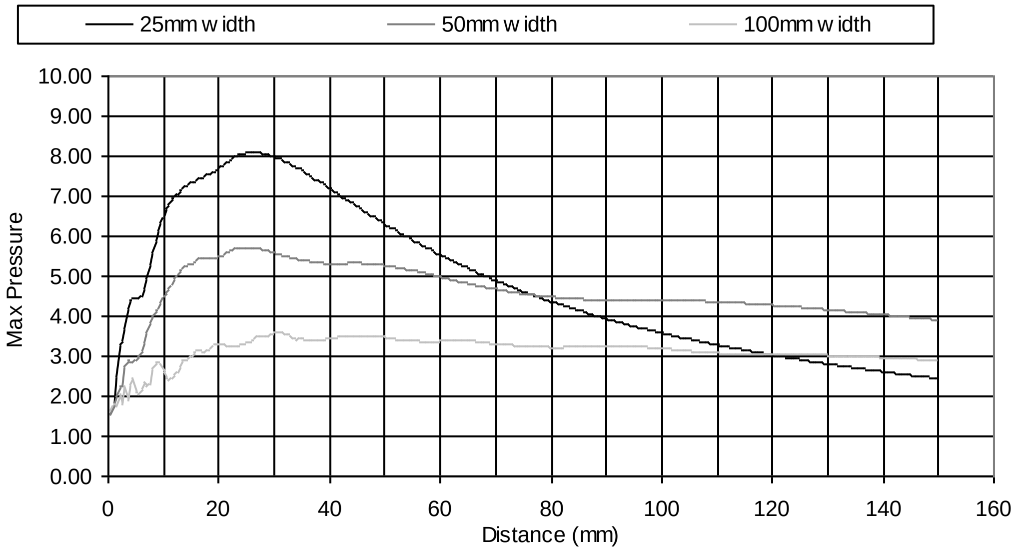


Figure 5 Maximum performances for 64 elements transducer with a number of different widths.

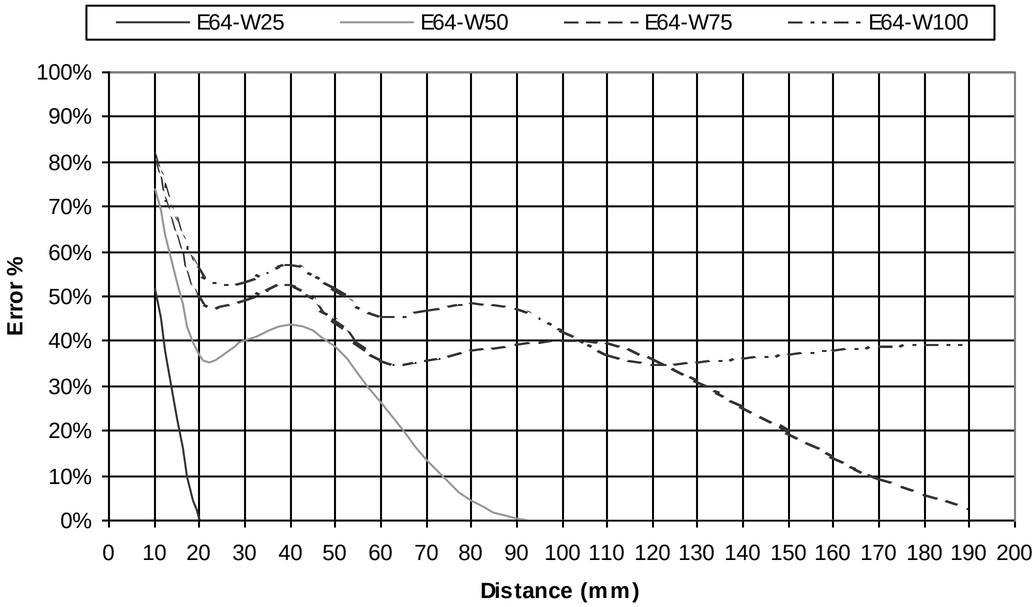
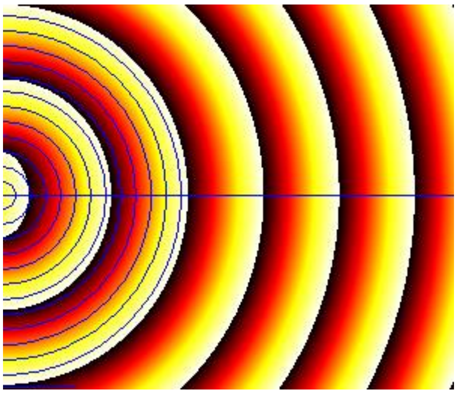
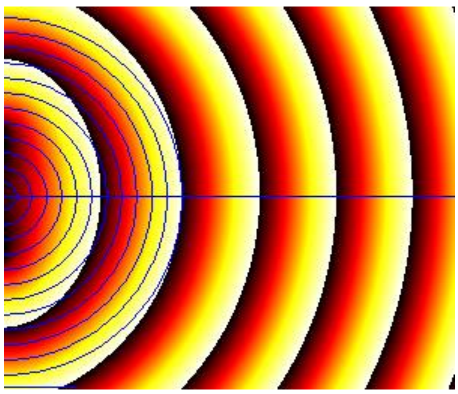


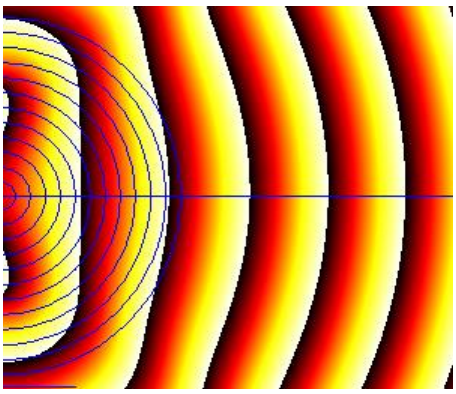
Figure 6 Percentage of underperformance against focal distance. 64 elements transducers with a range of different array widths.



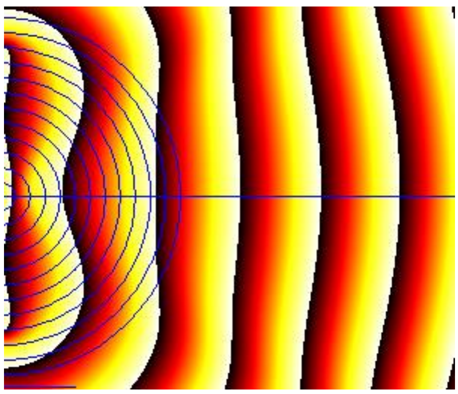
0.5 Wavelength



1 Wavelength



1.5 Wavelength



2 Wavelength

Figure 7 Phase pattern of array element with different element width

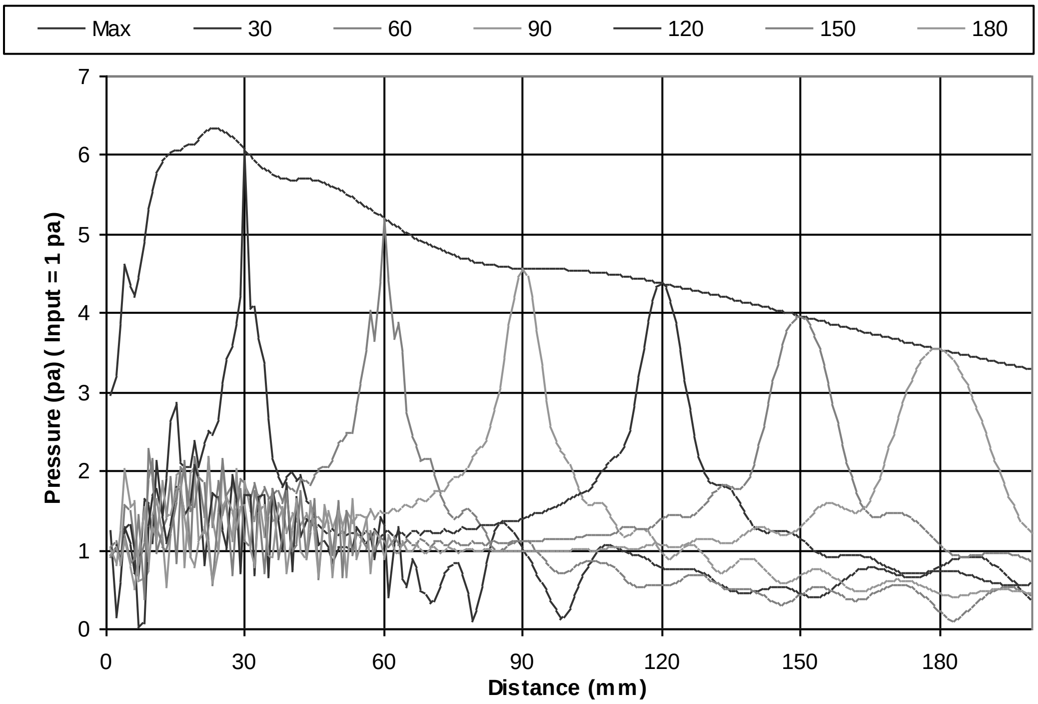


Figure 8 Pressure against distance from the transducer at different focal distance (64 elements, 50mm width)

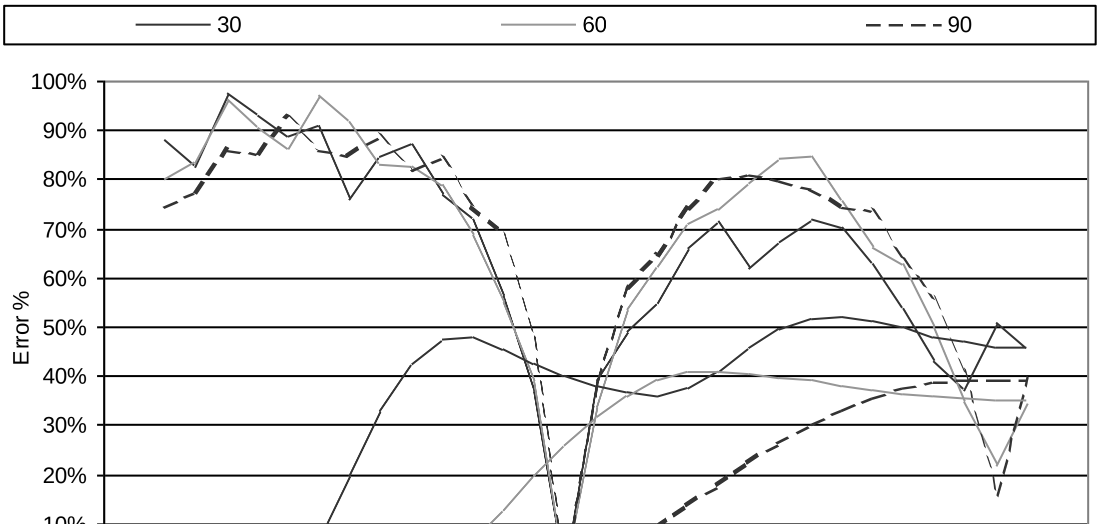


Figure 9 Percentage of error against frequency for difference focal distance at 30mm, 60mm, 90mm. (Advanced Focusing Equation and Generalised Focusing Equation)

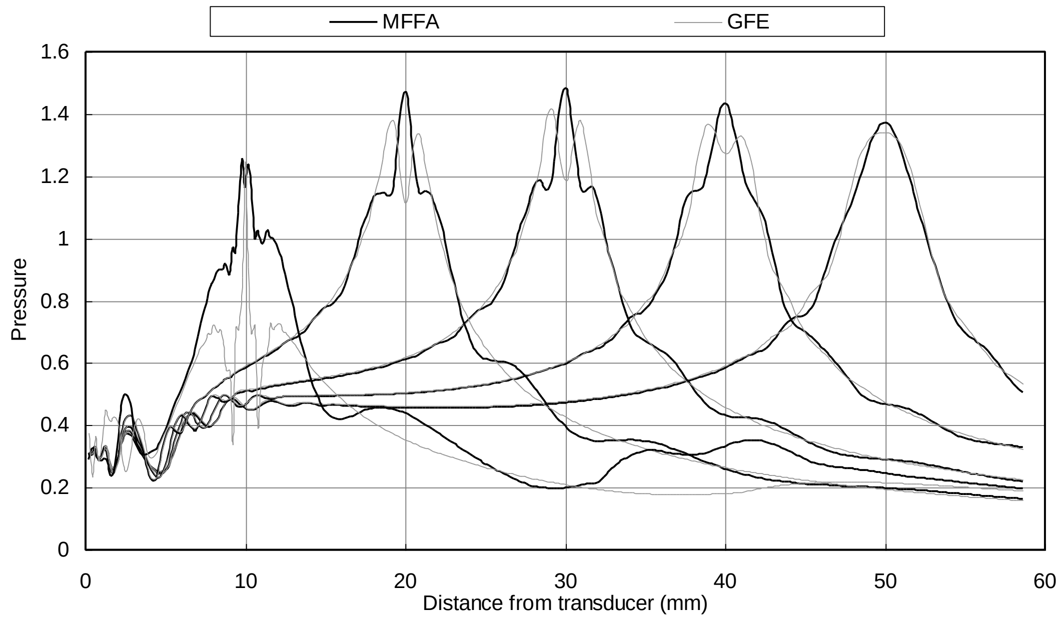


Figure 10 MFFA Vs GFE on Case 50mm width, 3mm height array transducer with 64 elements, centre frequency at 5Mhz, 3Mhz bandwidth, in water.

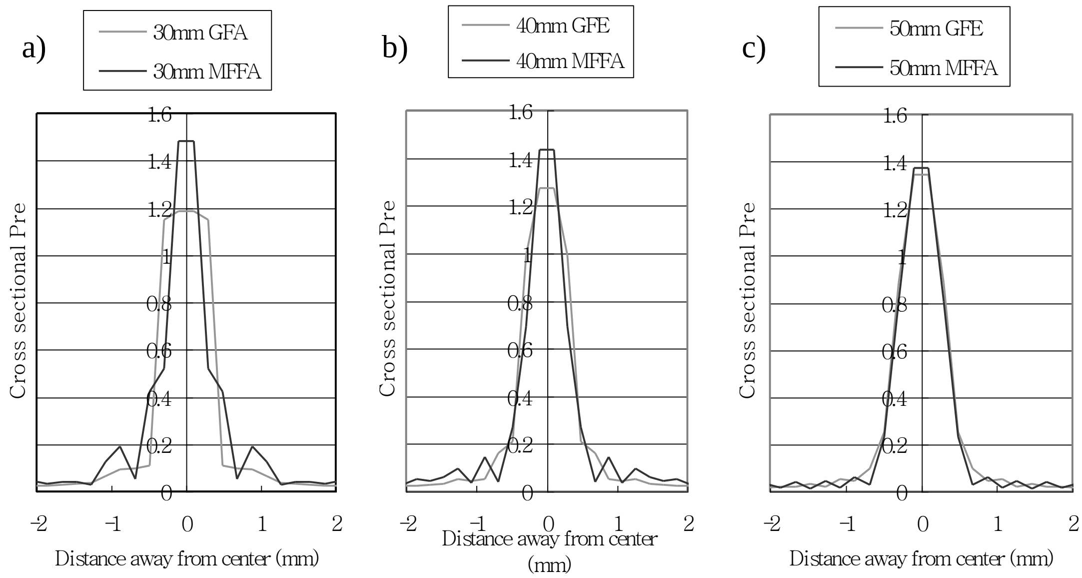


Figure 11 MFFA Vs GFE on Case 50mm width, 3mm height array transducer with 64 elements, centre frequency at 5Mhz, 3Mhz bandwidth, in water (cross-sectional on 30mm, 40mm, 50mm focusing).